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van Hiele Levels of Thinking Predict Students' Mathematics Grade

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ABSTRACT

The van Hiele levels of thinking has five reasoning levels, namely, holistic, analytic, abstract, deductive, and rigorous. This study aimed to determine the effects of spatial activities to the students' van Hiele Levels of thinking. It evaluated the van Hiele levels of geometrical reasoning taking into account the van Hiele level they reflected and their mathematical accuracy after exposure to spatial activities. Pretest-posttest design was used in this study. Sixty third-year high school students from five sections were the subjects with 30 students each in the control and experimental groups. The results revealed that only Level 1 in the post-test was significant. As to the type of reply, the post-test results showed that the control group acquired low acquisition to high acquisition in each level while the experimental group had low acquisition to complete acquisition in each level. Only Level 2 in the control group and Levels 3 and 4 in the experimental group could predict Mathematics grade. The control group had weaker reasoning capabilities in answering geometry problems; while the experimental group increased their level of reasoning, and thus, were able to answer geometry problems. This study concludes that the exposure to spatial activities would enhance the levels of reasoning of the third-year students in the study of geometry.

Keywords - Mathematics Education, Modified Van Hiele Levels of Thinking, Spatial exercises, Mathematics grade, True Experimental Pretest-posttest Design, Don Carlos, Bukidnon, Philippines

INTRODUCTION

Secondary school geometry turns out well when there is a solution for student difficulty with higher order thinking skills through the van Hiele levels theory (Usiskin, 1982). This theory says that a learner progresses through a sequence of five reasoning levels. However, students are not ready for a formal deductive geometry and they are of different levels (Mayberry, 1981).

In the present classroom environment, students are not anymore considered as a receiver and absorber of knowledge, but they are already taking an active part in explorations, investigations and discussion so as to build knowledge of their own. The ideas are borne in the student's mind and the teachers act as facilitators of learning. Students should be skillful in using higher order thinking skills (Mistretta, 1999).

Critical thinking, mathematical reasoning and proof are skills that a student should possess in all areas in mathematics. As a result, students may use mathematics as a tool in understanding and give meaning to the things and happenings around them (NCTM Standards). The National Council for Teaching Mathematics (NCTM) calls for greater emphasis on reasoning in all areas. Reasoning, then, would indicate a chance for success in proof-writing in geometry.

Geometry is one of the special subjects in secondary mathematics Curriculum. It has a special place due to a variety of concepts. It can be seen in other areas such as psychology because it represents the abstraction of visual and spatial experiences. This subject can also be integrated with other branches in mathematics because it provides approaches for problem solving, drawings, diagrams, coordinate system, vectors, transformations, and so on. Considering Euclid as the Father of Geometry, he was credited for the reduction of geometrical concepts to mathematical form which helped many mathematicians solve problems. Thus, it also leads to the development of critical thinking, mathematical reasoning, and proving abilities.

Geometry develops man's way of reasoning through proving statements. Students being poor in writing proofs is a problem of geometry teachers. It is imperative that geometric maturity among students is secondary to instruction to develop their levels of thinking (Crowley, 1987). One of the factors that might affect the proof-writing skills is the level of development of students. But gender has nothing to do with the acquisition of the levels (Halat, 2006). One topic in Geometry which is highly recommended by researchers to be used to determine the van Hiele levels is on triangle congruence and inequalities. The test on van Hiele theory should predict the performance of the students (Usiskin & Senk, 1990) and the activities should exemplify van Hiele theory that can convert into classroom practices (Teppo, 1991).

Lord's (1985) study revealed that there was an improvement in visuospatial cognition among students through spatial visualization. This was also supported by Clements et al. (1997) that there was a positive effect on spatial abilities. Thus, levels of thinking might be enhanced with spatial activities.

FRAMEWORK

This study was anchored on a theory on levels of thinking developed by two Dutch mathematics teachers, Pierre van Hiele and Dieke van Hiele-Geldof in the late 1950's. Based on their teaching and research, they observed that in learning geometry, a learner progresses through a sequence of five reasoning levels (Burger & Shaughnessy, 1986). Piaget's work on readiness influence much the van Hiele model. This model consists of five sequential levels of understanding by both instruction and maturity of the students. The following are the differentiated levels:

1. Holistic Level- when thinking primarily is holistic. Students use imprecise properties to compare drawings and identify shapes.
2. Analytic Level- when thinking analytically, students focus explicitly on properties or attributes of shapes. At this level, mathematical proof may be explicitly misunderstood and unappreciated.
3. Abstract Level- students are thinking abstractly from complete definitions that are applied explicitly. Definitions can be modified or used in equivalent forms.
4. Deductive Level- at this level, the mathematical structure of Geometry has completely emerged for the students. Thus, they can reason deductively within a particular mathematical system, although perhaps not realizing that different axioms would produce a different system, and hence a different theorem.

5. Rigorous Level- at this level, students appreciate the investigations of various systems of axioms and logical systems, also are able to reason in the most rigorous way within the various systems.

Senk (1989) studied 1,520 geometry students from five states and found that students had difficulty in writing proofs. Her study also included the van Hiele levels of reasoning. She reported that the higher the van Hiele entering level, the greater is the possibility that the students master proof-writing later in the year, and the lesser is the likelihood that he or she would fail to learn to write proofs. Level 2 appeared to be the critical entry level. A student beginner at level 2 had a 50-50 chance of mastering proof-writing by the end of the year. A student at Levels 3 or 4 had a significantly greater chance of mastering proof-writing.

Gutierrez et al. (1991), both from the Universidad de Valencia together with Fortuny (1991) of Universidad Autonoma de Barcelona had another method of examining the acquisition of the van Hiele Levels. Their study used spatial geometry test. In their research, they considered only the four van Hiele Levels, namely: Level1 (Recognition); Level 2(Analysis); Level 3 (Informal Deduction; and Level 4 (Formal Deduction). Each level consists of a degree of acquisition with the corresponding points: no acquisition, 0-15; low acquisition, 16-39; intermediate acquisition, 40-60; high acquisition, 61-85; complete acquisition, 86-100. Each item on the test has its type.

The modification used by Gutierrez et al. (1991), was adopted in the methodology of this study.

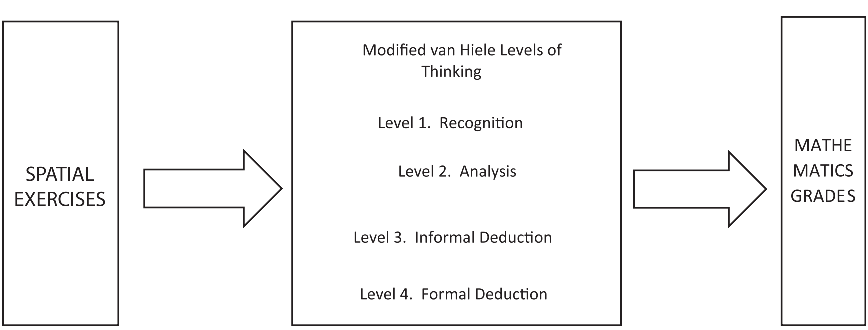


Figure 1. A Schematic diagram of the effects of spatial activities on the modified van Hiele Levels of thinking that predict mathematics grades

OBJECTIVES OF THE STUDY

Generally, this study evaluated the van Hiele levels of geometric reasoning and the students' answers taking into account the van Hiele level they reflected and their mathematics accuracy after exposure to spatial activities.

Specifically, this study aimed to: 1) determine if there is a significant difference in the van Hiele levels of thinking of students exposed to spatial activities and those not exposed; 2) determine the degree of acquisition of each van Hiele level of the students in relation to the type of reply they indicate at each level; and, 3) relate the van Hiele levels of thinking with the mathematics grade of the third-year students.

METHODOLOGY

The research was conducted in Bocboc National High School, Bocboc, Don Carlos, Bukidnon. The subjects of the study were the 60 third-year students with 30 boys and 30 girls. They were randomly assigned to the control or experimental group. There were fourteen sessions or exposures to spatial activities. The researcher obtained an informed consent from the respondents in compliance to research ethics protocol.

The instruments used in this study were the worksheets containing spatial activities and pre-test and post-test in van Hiele levels of thinking with the topic on Similarity of Triangles. The Spatial Exercises consisted of 29 items. Each item in the test had a corresponding van Hiele level of thinking. This was divided into fifteen activities. The 29-item exercises were subjected to item analysis and reliability test. The test was shown to the advisers for construct validity. This instrument was tried out. Using Chronbach alpha through SPSS software, the test has a reliability coefficient of 0.74 which indicates that it is reliable.

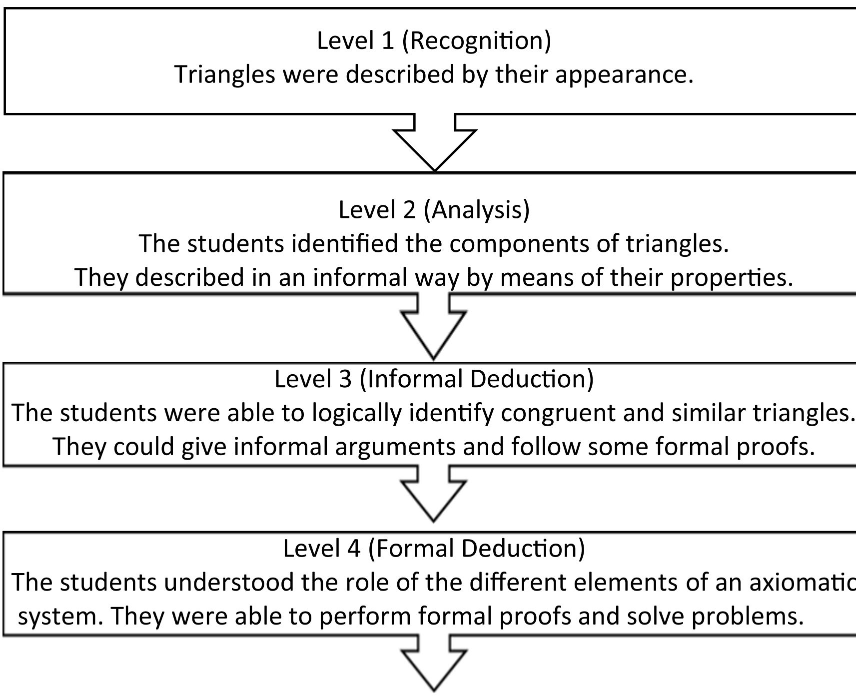
The Pretest and Posttest consisted of questions on triangles, following the van Hiele levels. There were five items in the test taken from the textbook and some were made by the researcher. These questions were shown to the advisers for construct validity. The revised test was tried in a New Nongnongan National High School for the first try-out. The items were subjected to item analysis. The Chronbach alpha reliability coefficient was 0.8017.

The lesson was based on triangle similarities that was taught in the third grading period. This was based from the textbook of the Department of Education, Culture and Sports (DECS). The lesson plans were copied from the

Philippine – Australia Science and Mathematics Education program teacher Support Manual Volume 2.

The measurement of van Hiele levels of thinking was patterned from the works of Gutierrez et al. (1991) who used three-dimensional geometry, specifically on spatial geometry. In this study, instead of using three-dimensional geometry, plane geometry was used.

The test design was based on the descriptors for the van Hiele levels 1 to 4, as follows:



After the levels had been identified, each answer was analyzed carefully as to what type.

Gutierrez et al. (1991), proposed an assessment procedure consisting a series of open-ended items and criteria for evaluating students' responses to each item, which were used in this study. Any reply to an open-ended item was assigned to one of the types of answer with the corresponding weight as shown in the following table.

Table 1. Weights of the different types of answers and their descriptors

Type	Descriptors	Weight
0	No reply or answers.	0
1	Answers indicated that the learners had not attained a given level and did not give information about any lower level.	15
2	Wrong and insufficient worked answers that gave some indication of a given level of reasoning; answers that contained incorrect and reduced explanations, the reasoning process or result.	20
3	Correct but insufficiently worked out answers that provided some sign of a given level of reasoning; answers that contained very few explanations, reasoning process or very incomplete results.	25
4	Correct or incorrect answers that clearly reflected the characteristic features of two consecutive van Hiele levels and that contained clear reasoning process and sufficient justification.	50
5	Incorrect answers that clearly reflected a level of reasoning; answers that presented reasoning process that were complete, but incorrect or answers that presented correct reasoning process that did not lead to the solution of the stated problem.	75
6	Correct answers that clearly reflected a given level of reasoning but that were incomplete and insufficiently justified.	80
7	Correct, complete and sufficiently justified answers that clearly reflected a given level of reasoning.	100

After assigning weights to each of answer, these numbers were used to determine the degree of acquisition of each level (Gutierrez et al., 1991).

Table 2. Quantitative and qualitative interpretation of the process of acquiring a level

Degree of Acquisition of Each Level	Qualitative Interpretation	Quantitative Interpretation
No Acquisition	Students were not conscious about the intense of or need for thinking methods, specific to a new level	0-15
Low Acquisition	Students were beginning to be aware of the method of thinking at a given level of their importance, and they were trying to use them	16-39

Intermediate Acquisition	Students were using the methods of the level more often continuously and accurately but lacked the mastery of these methods. There were confusing answers.	40-60
High Acquisition	Students had more experience, and their reasoning was progressively strengthened but made some mistakes.	61-85
Complete Acquisition	Students had complete mastery of this way of thinking and used it without difficulties	86-100

Extraneous variables may directly or indirectly affect the overall results of the study. Thus, the selection of the subjects of the study was carefully done through the use of appropriate sampling techniques. The students included in the sampling had a grade of 80% and above in Mathematics III in the second grading. However, there were only few boys who qualified. The researcher decided to include male students with a grade below 80% but with no failing grade. The two groups were exposed to the same lesson, approaches and time schedule. All groups were given the same type of test which were administered at the same time.

This study used these statistical techniques: frequency count, percent and weighted average, T-test, Chronbach alpha and multiple regression analyses. Statistical Packages for Social Sciences (SPSS) was used in the processing of data.

The duration of this study was only fourteen days which was very short with sixty students only. There was not enough time of exposing the experimental group to the spatial activities. The spatial activities were only planar without including the three-dimensional figures or objects. This could somehow contribute to elevating the van Hiele levels of thinking.

RESULTS AND DISCUSSION

There were thirty students each in the treatment groups who took the pre-test with the degree of freedom of 29.

There is no significant difference between the two groups in the pre-test at $p< 0.05$ because the two groups are matched. Under the first van Hiele level, they are both described to be under the degree of "high acquisition" (HA). There is no significant difference between the two groups in Level 2 with a mean difference of 6.32 and the probability of 0.799. They have no acquisition of this level. In

Level 3, there is also no significant difference between the two groups. The two groups show a low performance in writing proofs because there is no acquisition of Level 4.

After fourteen sessions of exposing the experimental group to spatial exercises, the results of the post-test showed that there is a significant difference between the two means in level 1. The experimental group does a remarkable performance in terms of recognition of types of triangles in the post-test. The control group has highly acquired (HA) Level 1 but the experimental group has completely acquired this level.

There was no significant difference between the means of the two groups in Levels 2, 3 and 4, though, the experimental group has higher means. This happened due to very short exposure of the spatial exercises. In terms of the degree of acquisition, the control group has "low acquisition" (LA), "Intermediate acquisition) (IA), and "low acquisition" (LA) in the three levels, respectively. Meanwhile in the experimental group, there was an "intermediate acquisition" in levels 2 and 3 and low acquisition in level 4.

Hence, only in Level 1, the two groups are significantly different in their means. The experimental group did a remarkable performance in terms of recognition of types of triangles in the posttest. There was no significant difference in the next 3 levels. However, the two groups' means differ numerically.

The result conforms to the study of Gutierrez et al. (1991), that the students had completely acquired Level 1. Clements et al. (1997) also supported the result of this study. The application of spatial thinking improves the abilities of the student in terms of understanding geometric concepts.

Multiple Regression Analyses of the Pre-test on the Modified van Hiele Levels of Thinking and Mathematics Grade

The resulting linear equations shown below describe the statistical relationship between the predictor variables (levels of thinking) and the response variable (mathematics grade).

That results revealed that among the four modified van Hiele Levels, only Levels 3 and 4 in the pre-test are the best predictors of the Mathematics grades in the control group and only Level 2 in the experimental group. These levels are statistically significant at $p < 0.05$.

The beta coefficients for levels 3 and 4 are 0.341098 and -0.427842, respectively, with a constant of 80.048747. Thus, the equation useful in predicting Y', mathematics grade of the control group, would be as follows:

$Y' = 80.0487 + 0.341098L_3 + -0.4278L_4$

While, in the experimental group, the beta coefficients of level in 0.187291 with a constant of 79.8318. Hence, the model in predicting Y', mathematics grade of the experimental group, would be as follows:

$Y' = 79.8318 + 0.187291L_2$

Multiple Regression Analysis of the Post-test of the Modified van Hiele Levels of Thinking and Mathematics Grades

The results showed that only Level 2 predicts mathematics grade of the control group in the post- test while Levels 3 and 4 are the best predictors of mathematics grades of the experimental group. These are statistically significant at $p < 0.05$.

The beta coefficient of Level 2 as the best predictor of mathematics grade in the control group is 0.059012 with a constant of 80.9583. Therefore, the equation which can be used in predicting Y', mathematics grade in the control group, would be as follows:

$Y' = 80.9583 + 0.059012L_2$

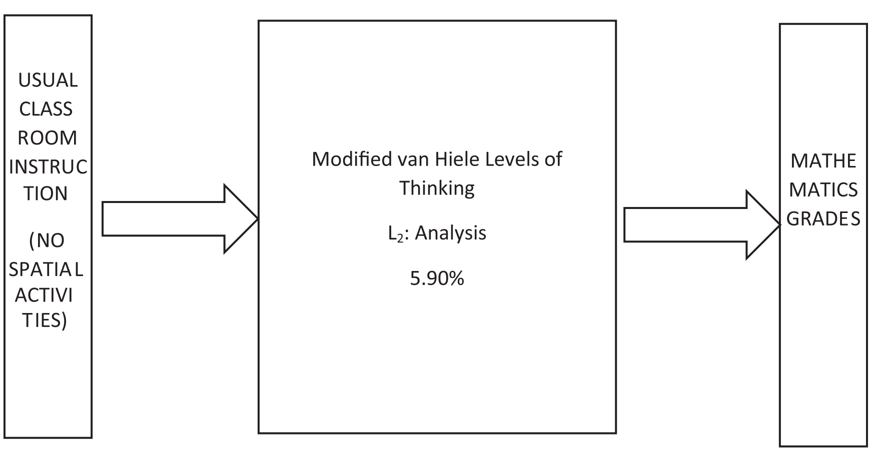


Figure 2. A Resulting Paradigm on the Modified van Hiele Levels of thinking that Predict Mathematics Grades of the Control Group

The best predictors of Mathematics Grades of the experimental group in the post-test which are Levels 3 and 4 have a beta weight of 0.065183 and 0.069685, respectively, with a constant of 81.564509. So, Level 4 has a greater effect to the mathematics grades of the experimental group than Level 3. Thus, the model which is useful in predicting Y', mathematics grade of the experimental group, would be as follows:

$$Y' = 81.584509 + 0.065183L_3 + 0.069685L_4$$

The aforementioned linear equations show that mathematics grade can be predicted through van Hiele levels of thinking.

Hence, after exposure to spatial activities, the levels of thinking elevated to Levels 3 and 4.

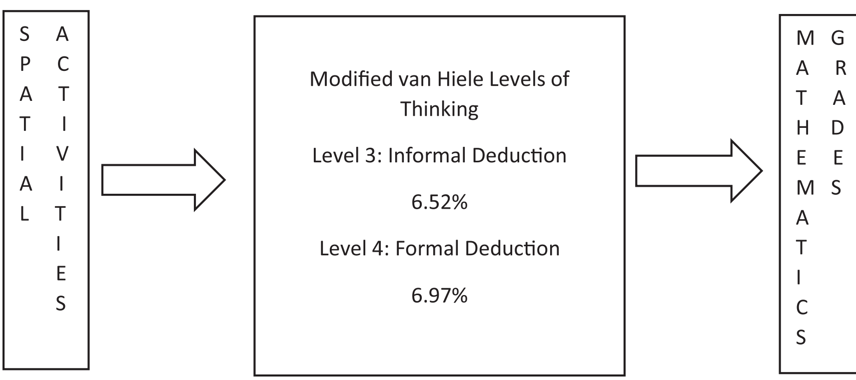


Figure 3. A resulting paradigm of the Modified van Hiele Levels of Thinking that Predict Mathematics Grades of the Experimental Group

CONCLUSIONS

Based on the findings of the study, the following conclusions were drawn: 1) Since there was no significant difference between the van Hiele Levels of thinking of the control and the experimental group in the pre-test, students were of the same level. In the post-test, spatial exercises worked effectively because it was significant in Level 1 and the mean of the experimental group were numerically higher than the control group in all levels ;2) Though both groups were on the same degree of acquisition in all levels, spatial activities worked effectively to the students because the experimental group outnumbered the control group in all levels; 3) The van Hiele levels of thinking could predict mathematics grades of the students; and 4) Exposure to spatial activities would enhance the levels of reasoning of the third-year students in the study of Geometry.

TRANSLATIONAL RESEARCH

If the student's level of thinking is known, this study is very useful to the parents for them to provide activities at home that would enhance thinking. This would also be beneficial to curriculum developers to design a curriculum where thinking skills are given more emphasis. A mathematics teacher in the high school can also derive advantage from this study and can lead them to craft appropriate classroom activities on proof-writing and problem-solving that will help improve thinking skills. In effect, the students' levels of thinking and mathematics grade can be improved through a concerted effort of the aforementioned significant individuals. More importantly, the Department of Education can collaborate with experts in mathematics education, mathematics teachers and computer programmers to create workbooks or a software where students can interact to improve their levels of thinking. Research institutions can also join forces with researchers in basic and applied mathematics to find new interventions or strategies in which their findings can support in the improvement of the levels of thinking of students.

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