DOI: https://doi.org/10.7719/872

Mathematical Misconceptions of Senior High School Students: Implications to Mathematics Curriculum

KAREN A. QUINIO[1], POLEMER M. CUARTO[2]
[1]Domingo Yu Chu National High School, Oriental Mindoro, Philippines
[2]Mindoro State University, Oriental Mindoro, Philippines
ORCID No.: Karen A. Quinio https://orcid.org/0009-0005-4082-7576
Polemer M. Cuarto https://orcid.org/0000-0002-5507-3640

Corresponding author: karen.quinio@deped.gov.ph

Originality: 100% Grammarly: 98% Plagiarism: 0%

ABSTRACT

Article history:

Received: 20 Mar 2023
Revised: 28 Aug 2023
Accepted: 10 Oct 2023
Published: 31 Oct 2023

Keywords — general mathematics, misconceptions, modular learning modality, statistics and probability, senior high school students

Misconceptions often pose significant barriers to effective mathematics learning. This study aimed to investigate the misconceptions prevalent in general mathematics and statistics and probability among senior high school students taught during the modular learning modality. Through a mixed-methods approach, involving quantitative surveys and qualitative interviews, mathematical misconceptions such as procedural knowledge, misapplication of formulas, internal barriers, misleading assumptions, and limited question understanding were identified. The study highlights the implications of these findings for curriculum design, teacher professional development, student-centered instruction, and the development of supplementary resources. Moreover, the study underscores the need for tailored interventions to rectify misconceptions,

cultivate a deeper understanding of mathematical concepts, and foster accurate probabilistic reasoning among senior high school students during the new normal mode of instruction.

INTRODUCTION

Senior high school mathematics education is at a critical point, with an urgent need to help students address misconceptions that can greatly hinder their understanding of essential mathematical concepts. Following the onset of the pandemic and the introduction of the modular mode of learning, a notable learning gap in mathematics emerged among students in the Philippines, with misconceptions being a prominent factor. This situation renewed the urgency to address learning gaps and recovery efforts. Misconceptions from prior lessons can hinder the understanding of new topics, a phenomenon equally applicable to senior high school students studying mathematics, thus can hinder students' learning progress (Chian, 2020).

Errors and misconceptions in mathematics learning are distinct yet interconnected concepts. Errors typically result from mistakes and carelessness, while misconceptions stem from misunderstandings, making it crucial to distinguish between them (Voon et al., 2017). Recent empirical studies have explored how students develop these misconceptions, categorizing them into systematic, random, and thoughtless errors (Makonye & Khanyile, 2015). Systemic mistakes arise from misunderstandings of core concepts, whereas random errors lack clear patterns (Ang & Shahrill, 2014; Schnepper & McCoy, 2013).

In the global setting, the prevalence of errors and misconceptions extends beyond mathematics and includes other subjects (Aliustaoglu et al., 2018; Burgoon et al., 2017; Mohyuddin & Khalil, 2016). Several factors contribute to their formation, including student attitudes toward math, teaching methods, preconceptions, limited understanding, inadequate modeling, and underdeveloped higher-order thinking skills (Blazar & Kraft, 2017; Diyanahesa et al., 2017; Kusmaryono et al., 2020; Saputri & Widyaningrum, 2016; Skott, 2019). Mathematical misconceptions often develop from generalized prior knowledge, causing students to believe their incorrect methods are correct or to feel uncertain about their approaches (Im & Jitendra, 2020; Neidorf et al., 2020; Rushton, 2014). Errors can also result from incompetence or a lack of careful checking (Hansen et al., 2014). These misconceptions hinder understanding, lead to repeated errors, and negatively impact academic performance and attitudes (Belbase, 2013). Educators must address these misconceptions and ensure a firm grasp of fundamental concepts before introducing new topics (Tippett, 2010;

Sarwadi & Shahrill, 2014; Ocal, 2017). Various misconceptions have been observed in mathematics education, emphasizing the need to rectify and clarify these issues (Hayati & Setyaningrum, 2019).

In the Philippines, a critical issue emerged from the National Achievement Test results for Grade 12 students in the 2017-2018 school year, with the Bureau of Education Assessment (BEA) reporting a National Mean Percentage Score for Mathematics as low as 29.60, falling below the national standard and ranking as the lowest among the seven subject areas assessed. This underscores the urgent need to investigate the factors contributing to this low performance and to implement interventions aimed at improving the mathematical achievement of Grade 12 students (Makonye & Khanyile, 2015). In a broader context, the inadequate mastery of fundamental math concepts significantly increases the likelihood of errors and misconceptions in problem-solving. A strong foundation in mathematics is paramount for effective learning, particularly for senior high school students, where existing misconceptions may impede their progress. Detecting and addressing these misconceptions is essential for educators to provide additional support to those students who require it and ensure a clear starting point for their lessons (Makonye & Khanyile, 2015).

While some studies have explored misconceptions in mathematics education globally, there is a noticeable gap in research focusing specifically on senior high school students in the Philippines. Existing literature tends to provide a broad overview of misconceptions in mathematics or focuses on specific grade levels, neglecting the unique context of senior high school education. Understanding the extent and nature of misconceptions among senior high school students in the Philippines is crucial, as it could shed light on targeted interventions and instructional strategies to enhance mathematical comprehension within the context of senior high school education, especially those who are products of the modular learning modalities during the pandemic times. Addressing this gap would contribute valuable insights to both educators and researchers, facilitating more effective teaching and learning approaches in Mathematics. It could provide implications for the senior high school mathematics curriculum in the Philippines.

FRAMEWORK

This study is anchored on two theories: Constructivist Learning Theory and Conceptual Change Theory. Mathematical misconceptions among senior high school students often stem from their incorrect prior beliefs and ideas in mathematics, which they may consider as true. These misconceptions are rooted in the principles of constructivist learning theory. According to this theory, learning is an active and contextualized process of constructing knowledge based on personal experiences rather than passively acquiring it. Furthermore, the Conceptual Change Theory offers valuable insights into how students develop and modify their conceptual understanding. This theory suggests that learners often hold preconceived ideas or misconceptions, and effective learning involves recognizing and challenging these misconceptions. The process of conceptual change requires students to restructure their mental models and replace misconceptions with accurate concepts. In the context of this study, identifying and addressing misconceptions among students is central to the application of both constructivists learning theory and conceptual change theory.

In essence, misconceptions may arise due to the teaching and learning of mathematics in a constructivist manner. To identify and address these misconceptions, it is imperative to first define the students' prior knowledge regarding mathematics. This understanding of their existing knowledge structures serves as a crucial means to identify and rectify their mathematical misconceptions effectively. The interplay of these two theories—constructivist learning theory and conceptual change theory—provides a comprehensive framework for addressing and preventing mathematical misconceptions among students.

OBJECTIVES OF THE STUDY

The primary objective of the study was to investigate the prevalence and nature of mathematical misconceptions among senior high school students. Specifically, the study aimed to (1) identify and categorize the prevalent mathematical misconceptions of senior high school students and (2) propose strategies and interventions to enhance the senior high school mathematics curriculum to address these misconceptions.

METHODOLOGY

Research Design

The research design employed in this study was an Explanatory Sequential Mixed Method Design, which involves collecting and analyzing both quantitative and qualitative data sequentially (Creswell & Plano, 2018). This design is characterized by an initial quantitative phase followed by a qualitative phase, with data integration at the end. The first phase focused on quantitative data collection through a mathematical misconception test, while the second phase involved qualitative data collection through interviews.

Participants

Non-probability sampling, specifically convenient sampling, was utilized to select participants. The study included senior high school students from various regions in the Philippines during the 2022-2023 school year, including the National Capital Region. In the initial phase, 80 senior high school students participated in the mathematical misconception tests. For the subsequent phase, 20 respondents were purposively chosen from those who had exhibited the highest number of mistakes in the initial test.

The researcher opted to focus on senior high school students to identify and rectify misconceptions, as these students were still studying high school-level mathematics, making it easier to address any misconceptions. Moreover, mathematics was identified as a subject prone to misconceptions, so addressing these issues while students were in secondary education allowed them to recognize and correct misconceptions early. This, in turn, would better equip them to solve mathematical problems effectively and pursue higher-level math courses. The study took place in various secondary schools across different regions in the Philippines, and the participants were senior high school STEM students from various educational institutions who could provide the necessary data for the researchers.

Instrumentation

Two research instruments were employed for data collection. In the first phase, a Mathematical Misconceptions Test was used, compiled from various studies, and tailored to the study's topic. This test covered general mathematics, probability and statistics, with reliability confirmed through McDonald's Omega reliability analysis (Pfadt et al., 2022). Its pairwise complete cases have a reliability measurement of 0.862 which indicated a good reliability. The second instrument was interview questionnaires formulated based on first-phase data. Three experts in the field validated these questionnaires.

Data Collection and Analysis

This study used an innovative method for identifying misconceptions called the "Scaling Individuals and Classifying Misconceptions (SICM)" model (Bradshaw & Templin, 2014). This is an innovative approach that combines Item Response Theory (IRT) and Diagnostic Classification Models (DCM) to assess misconceptions in educational contexts. Instead of measuring traditional skills, the SICM model uses categorical latent variables to represent misconceptions in students and categorizes individuals based on their misconceptions. Using the SICM Model, the participants in this study were subjected to a two-phase data collection process. In the initial phase, participants were administered a mathematical misconception test, which was designed to gauge their understanding of various mathematical concepts. Additionally, they were asked to respond to open-ended questions that probed their comprehension of commonly misunderstood topics in mathematics. The purpose of this phase was to obtain quantitative and qualitative data that could shed light on the extent and nature of misconceptions held by the participants.

In the subsequent phase, participants who had completed the initial test and open-ended questions were selected for interviews. These interviews were structured around the responses provided by the participants in the first phase. The interviews aimed to delve deeper into the participants' thought processes, allowing them to explain and clarify their responses further. This phase provided valuable qualitative insights into the underlying reasons for the misconceptions identified.

The quantitative data extracted from the mathematical misconception tests were subjected to a thorough analysis utilizing descriptive statistics, such as frequency, percentage, and mean calculations. This quantitative approach offered a comprehensive overview of the prevalence and magnitude of misconceptions within the study's participant group.

Simultaneously, the qualitative data collected from the open-ended questions and interviews underwent content analysis. This involved systematically reviewing and categorizing the participants' responses to pinpoint and define specific misconceptions. Through the content analysis, the researchers sought to gain a nuanced understanding of the precise nature and details of the misconceptions prevalent among the study's participants. This mixed methods approach provided a comprehensive assessment of the misconceptions in the field of senior high school mathematics education.

Research Ethics Protocol

Ethical concerns were addressed throughout the study. Participants received a consent letter detailing the study's purpose, its potential impacts, and the use of findings. This informed their decision to participate, and their choices were respected. Confidentiality and anonymity were upheld in accordance with the Data Privacy Act of 2012. The researcher safeguarded participant identities and shared data from the survey and questionnaires.

RESULTS AND DISCUSSION

Mathematical Misconceptions on General Mathematics

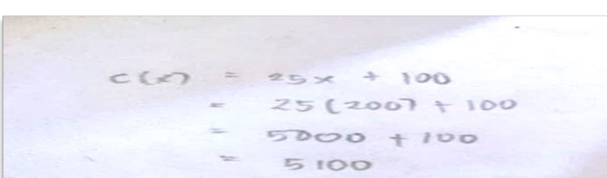
Table 1
Prevalent Misconceptions in General Mathematics Among Senior High School Students

Misconceptions	Frequency	Percentage	Rank
Absence of Meaning	64	80%	2
Misapplication of Formula	48	60%	3
Lack of Procedural Knowledge	68	85%	1
Internal Barrier	20	25%	4

*Absence of meaning***.** The first question in the general mathematics section aimed to assess participants' ability to solve problems involving functions. Specifically, the question presented a scenario where Kyla sells snacks, and her daily expenses were modeled by the function C(x) = 25x + 200, where x represents the number of snack items prepared. Participants were tasked with determining thier expenses for preparing 100 snack items.

The data indicated that only 60% of participants arrived at the correct solution. A substantial number of participants demonstrated a misconception referred to as the "absence of meaning." Figure 1 displays the distribution of responses collected from the participants.

Figure 1
Students Responses in Solving for the Value of C(x)



Among the 20 interview participants, seven of them, namely participants 3, 4, 7, 10, 11, 13, and 15, provided solutions similar to that shown in Figure 1. It was evident that these participants incorrectly represented the given information, leading to an erroneous solution. This observation highlights a lack of awareness among participants regarding the proper incorporation of the provided details within the equation. This shows the concept of “absence of meaning” as a prevalent misconception in this domain. This phenomenon aligns with the findings of Voon et al. (2017), who concluded that misconceptions related to the absence of meaning manifest as misunderstandings or incorrect applications of ideas, concepts, theories, or formulas within given equations.

A participant’s response during the interview further reinforces this identified misconception:

> Interviewer: Why did you select 200 as the value for the variable x instead of 100?
>
> Participant 7: *Honestly, I wasn’t sure how to substitute the value of x. While solving using the substitution method, I arrived at an answer, which I then used as my final solution.*

This response distinctly reveals that the participant struggled to grasp the underlying meaning of the problem statement. Instead, they proceeded to substitute an incorrect value into the equation. While the participants’ efforts to comprehend problems involving functions are commendable, this approach can inadvertently foster misconceptions such as the “absence of meaning.” This misconception emerges when individuals encounter functions that extend beyond the confines of pure mathematics, showcasing the importance of cultivating a deep understanding of the contextual meaning behind mathematical concepts.

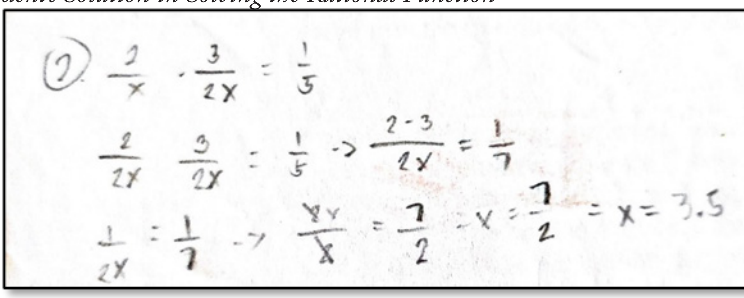
Misapplication of formula. The second question in this section involved solving a rational function: 2/x - 3/2x = 1/5, aiming to find the value of x. The results indicated that 80% of participants correctly solved the equation. However, a notable misconception emerged during the interviews, known as the “misapplication of formula.”

This misconception was particularly evident in the response of Participant P19. Inconsistency arose in the application of the equated function, notably when the fraction 1/5 was erroneously replaced with 1/7. Consequently, the participant’s solution led to an incorrect answer. Below is an excerpt from the interview, along with Figure 2, illustrating the participant’s solution.

Interviewer: How did you determine the value of x in the function 2/x - 3/2x = 1/5?

P19: *I started by finding the least common denominator (LCD) and proceeded with the solution. However, I made an error along the way, resulting in an answer of 7/2 instead of the correct 5/2.*

Figure 2
Student's Solution in Solving the Rational Function

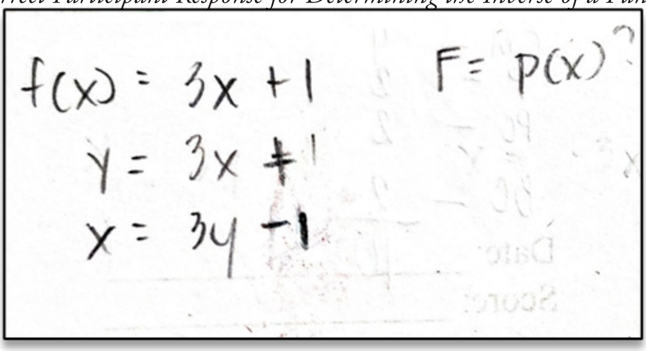


In the initial steps, the participant displayed an accurate grasp of obtaining the least common denominator (LCD). However, an inadvertent switch of values during the process led to an error in the final answer. Several research studies have explored misconceptions associated with the misapplication of formulas when solving rational functions. Özgür and Gürel (2018) examined high school students and identified how misconceptions often stem from inaccurately applying formulas or utilizing incorrect rules when manipulating rational expressions. A similar trend was observed in the study conducted by Şahin and Soykan (2016), showing that students frequently misapply formulas when dealing with rational equations and functions, leading to misconceptions and incorrect solutions. These studies have significantly contributed insights into the specific misconceptions arising due to formula misapplication within the scope of rational functions.

Lack of procedural knowledge. The question centered on determining the inverse of a function: if p(x) is the inverse of f(x) where f(x) = 3x + 1, what is p(x)? Remarkably, only 15% of the eighty (80) participants arrived at the correct answer. A prevailing misconception emerged during interviews, identified as the "lack of procedural knowledge."

Several interview participants, namely P3, P7, P9, P16, and P18, exhibited an incorrect approach by believing that interchanging the signs and variables was the sole procedure in determining the inverse of a function. They overlooked the essential step of solving for the value of y. Figure 3 displays an incorrect response from one participant, showing the extent of this misconception.

Figure 3
Incorrect Participant Response for Determining the Inverse of a Function



This student held the misconception that the inverse function is derived by merely changing signs, which is a flawed understanding. Moreover, the student prematurely stopped the process after the sign interchange, unmindful to the subsequent requirement of solving for y. This misconception distinctly aligns with the "lack of procedural knowledge." Additional interview responses further underscored this misconception:

> Interviewer: How did you arrive at the answer p(x) = 3y - 1 for the inverse of the function f(x) = 3x + 1?
>
> P3: *My understanding was to swap f with p and then consider the counterpart of 3x as 3y, leading to 3y - 1. I also reversed the sign from positive to negative.*
>
> *Interviewer: So, you interchanged variables but didn't proceed to solve for y, is that correct?*
>
> P3: *Yes, I wasn't aware of the subsequent steps in solving for the inverse of a function.*

Similar misconceptions were also observed in other responses:

> P7: I forgot the formula for finding the inverse of a function. I only changed the sign.
>
> P9: I believed that finding the inverse involved interchanging x and y, and then changing the negative sign to positive.
>
> P16: My understanding was to change the positive sign to a negative sign in finding the inverse.

The collective responses of these students clearly reveal their erroneous grasp of the process for solving inverse functions. This shows the strong association of this misconception with a "lack of procedural knowledge."

An in-depth exploration into the misconceptions surrounding the inverse function-solving process aligns with Kumar's (2018) study, which targeted the misconceptions prevalent among high school students during inverse function-solving. The study's findings clarified a notable deficit in procedural knowledge, leading to misguided problem-solving approaches. Students commonly faced difficulties in accurately recognizing and applying the required steps for determining function inverses.

Aligned with these findings, this paper stresses the significance of providing students with explicit, step-by-step procedural guidance. Such guidance facilitates the development of procedural knowledge, which is essential for the successful solution of inverse function problems.

Limited understanding of exponential functions. Exploring the understanding of exponential functions among students revealed a prevalent misconception, as demonstrated by the low accuracy rate of 15% in response to the question, "How do you understand exponential function?"

One question probing students' grasp of the meaning of exponential functions identified a significant misconception: their limited understanding of the concept. Exponential functions should serve as a foundational notion within general mathematics, especially among senior high school students. However, a closer examination of student responses revealed that their comprehension remained confined to "functions with exponents." Among the 20 interview participants, only three (P1, P4, and P9) consistently displayed this shallow understanding. Their responses exemplify this limitation:

Interviewer: How do you interpret the concept of an exponential function?

P1: *I equate exponential functions with functions involving exponents.*

Interviewer: But what if the exponent in a function is a variable, not a constant? Would it still be considered an exponential function?

P1: *No.*

Interviewer: How would you define an exponential function?

P4: *A function that employs exponents.*

P9: *I struggle to articulate its definition, but I know that if an exponent is present, the function qualifies as exponential.*

These responses expose a partial comprehension of exponential functions. Notably, P1 seemed unaware that an exponential function could involve an exponent that extends beyond mere constant, encompassing variables as well. This misconception is shown in the responses of P4 and P9.

This finding is similar to the study conducted by Chen and Eslami (2019), which investigated students' understanding of exponential functions through interviews and written assessments. Their findings showed a prevalent challenge among students in grasping the fundamental concepts associated with exponential functions.

This shows the significance of emphasizing basic mathematical concepts, such as the definition of exponential functions. Thorough discussions are important to prevent misconceptions from potentially influencing students' future mathematical endeavors. Moreover, mathematics teachers can lessen the development of misconceptions that may resurface in students' future mathematical encounters by addressing these foundational concepts comprehensively.

Internal barrier. The question focused on understanding the behavior of the graph of y = $\log_{\frac{1}{2}} x$ and presenting a table of values. The results revealed a distinct misconception termed "internal barrier." Among the 20 participants, two (2) displayed this particular misconception.

Understanding graph behavior was most significantly hindered by the presence of internal barriers, representing a noteworthy factor underlying misconceptions. Students dealing with internal barriers often experienced stress and difficulties when solving mathematical problems and encountered challenges in comprehending graph behavior. One of the misconceptions emerging from internal barriers is illustrated below:

Interviewer: What can you infer about the behavior of the graph of y = $\log_{\frac{1}{2}} x$? Before discussing your answer's basis, how do you generally perceive graph behavior?

P3: *Graph behavior involves determining if it trends upward or downward. However, in this context, I'm struggling since I lack the understanding of how to derive values for this particular equation.*

Interviewer: So, does this imply that when logarithmic functions are part of the equation, assigning values becomes a challenge for you?

P3*: Yes.*

The participant's response illustrates a state of confusion when dealing with the graph behavior of logarithmic functions. For some learners, understanding the behavior of graphs of logarithmic functions proves challenging due to internal barriers. Visualizing and interpreting logarithmic functions can indeed pose difficulties, as evident in the participant's response. This is similar to the findings of the study conducted by Ocak and Bayazıt (2017), which identified a range of misconceptions related to logarithms. These misconceptions encompassed aspects such as misconstruing the logarithmic function and dealing with its accurate graph representation.

The findings herein align with those of Ocak and Bayazıt (2017), indicating a shared challenge in comprehending logarithmic functions and their corresponding graph behavior. Addressing these internal barriers and promoting a comprehensive understanding of logarithmic functions within a supportive learning environment can contribute to dispelling such misconceptions and fostering effective mathematical comprehension.

Mathematics Misconceptions on Statistics and Probability

Table 2
Prevalent Misconceptions in Statistics and Probability Among Senior High School Students

Misconceptions	Frequency	Percentage	Rank
Lack of Procedural Knowledge	45	56.25%	3
Internal Barrier	72	90%	1
Lack of Understanding of the Question	71	88.75%	2
Misleading Assumption	30	37.5%	4

Lack of procedural knowledge. The question presented respondents with a scenario involving rolling two dice simultaneously and asked them to identify the event with a higher likelihood of occurring. Remarkably, all participants displayed misconceptions when grappling with this problem. Notably, 32 participants fell into the misconception of assigning a 'first die' and 'second die' result, erroneously overlooking the simultaneous nature of the dice roll. One respondent's response shows this misconception:

Interviewer: How did you arrive at the answer 36?

P3: *It's 6 multiplied by 6. Because 6 is... wait, am I right? That's what I thought, hmm, 6 factorial, and the other one is 2^6. Oops, I think I made a mistake. I'm confused about whether 5 or 6 should be the first or second result. Then, I realized that you can't determine which is the first outcome, whether it's the one on the left or right side. Am I right?*

In this instance, Participant 3 demonstrated uncertainty and quickly shifted answers upon realizing their initial response was incorrect. Furthermore, the participant incorrectly believed that the outcomes of rolling two dice could be designated as 'first' and 'second' results. This misconception aligns with the concept of "lack of procedural knowledge." This involves procedural errors stemming from an inability to execute manipulations or algorithms despite understanding the underlying concepts (Arum et al., 2017).

This finding corroborates the findings from Arum et al. (2017) study, emphasizing the importance of addressing the lack of procedural knowledge.

Remedying this misconception requires explicit instruction to bridge the gap between conceptual understanding and procedural execution.

Internal barrier. In addressing the question related to choosing a committee with two members from a pool of 10 candidates, responses revealed an internal barrier to understanding probabilistic problems. The responses demonstrated that probabilistic problem-solving was not straightforward for senior high school students. This observation was coupled with unfamiliarity or forgotten concepts from subtopics, possibly taught during their earlier educational years. A tendency to guess without proper problem-solving was also observed:

> Interviewer: Do you have an initial understanding of the topic covered in this questionnaire?
>
> P5: *Yes, I do, but it's not vivid in my memory. I can't recall it properly unless I revisit it.*

Participant 5's response indicated that while she possessed some initial familiarity with probability-related topics, the specifics had faded from memory and required revisiting. Humphrey and Masel (2014) shed light on the concept of outcome orientation as a misconception in probability. This misconception arises when individuals treat the possibility of an event happening or not happening as a confirmation of certainty rather than a measure of likelihood. Outcome orientation can manifest when individuals don't perceive the likelihood of an event as a prediction or probability (Gorham et al., 2019).

The findings show the impact of internal barriers, including unfamiliarity, forgotten concepts, and the tendency to guess, on students' ability to understand probabilistic problems. These internal barriers hinder their holistic understanding of probability concepts. Addressing these barriers through targeted instruction and support is important for guiding students toward a more accurate and effective understanding of probabilistic reasoning.

Lack of understanding of the question. Questions related to probability posed a substantial challenge for students, underscoring a notable factor: a lack of understanding of the question. The question involving the concept of the Heuristic of Availability addressed the probability of selecting committee members from a pool of ten candidates. Responses showed a struggle among students to discern the appropriate formula or solution for solving the problem. Additionally, comprehending the broader context of the problem proved challenging:

Interviewer: How did you derive 45 possibilities for two members out of 10 and 210 possibilities for 8 out of 10?

P2: *I'm not sure. Honestly, I resorted to guessing the answer to this one. To be honest, this is a recurring issue for me. I encounter difficulty comprehending word problems.*

The interviewer explains the problem.

P2: *I still don't understand.*

Interviewer reveals the answer

P2: *I'm still confused. My thinking was similar to fractions – if I choose 2, then 8 remain, and if I choose 8, then 2 remain. How did it end up being the same?*

As the participant repeatedly expressed, word problems of this nature presented significant comprehension challenges, leading to an incorrect response. P2's inability to explain their initial answer highlighted this struggle. These findings point to a potential difficulty in bridging the gap between students' intuitive grasp of probability, expressed in everyday language, and the specialized vocabulary and concepts employed in academic probability discussions.

This situation presents an opportunity to facilitate the transition from informal, intuitive probability understanding to a more formalized comprehension. The discrepancy between these two realms emphasizes the importance of employing diverse strategies to guide students from informal, intuitive probability knowledge toward a deeper, formal understanding. Addressing this gap can enhance students' problem-solving capabilities and build a stronger foundation for advanced probabilistic reasoning (Batanero et al., 2017).

*Misleading assumptions***.** Problem 4 pertains to the concept of Representativeness and falls under the category of Misleading Assumptions. It presents senior high school students with a scenario where they must decide who has a greater chance of winning in a lotto game – Camille, who chose consecutive numbers, or Sally, who selected random numbers. Remarkably, only 11.25% of participants answered correctly. The majority of those who chose Sally as the likely winner held a misleading assumption that random numbers are more likely to secure victory in a lotto game compared to consecutive numbers. An interview excerpt underscores this misconception:

Interviewer: Who do you think has a greater chance of winning, Camille or Sally?

P7: *I believe it's Sally. In a lotto, we usually pick random numbers, unlike Camille, who chose consecutive ones. That's why I picked Sally.*

Participant 7 demonstrated a presumption that random numbers are more likely to be drawn in a lotto game. This assumption, however, could be misleading. While it is true that random number generation seeks to minimize patterns and sequences, such as consecutive numbers, their occurrence in random selections is not impossible. Consecutive numbers can still emerge in random outcomes and be legitimate winning combinations. Assuming that random numbers in a lotto game predominantly exclude consecutive numbers may not accurately reflect the actual probabilities involved.

This finding was supported by the findings of Sharna et al. (2021) which emphasized that lotteries, much like other games of chance, center on the element of randomness. Random events unfold without a predetermined pattern or deliberate selection, implying that each number boasts an equal likelihood of being chosen as a winning number in an unbiased, separate drawing. This serves as a reminder that while patterns may be minimized, the element of chance still reigns supreme in lotto games, defying misleading assumptions about the prominence of consecutive numbers.

Proposed Strategies and Intervention to Enhance the Senior High School Mathematics Curriculum

Various strategies and interventions enhancing the Senior High School curriculum can address the issue of mathematical misconceptions and promote effective learning.

Table 3
Proposed Enhancement on the Senior High School Mathematics Curriculum

Proposed Strategies and Intervention	Activities
Explicit Misconception Addressing Modules	Integrate modules within the curriculum that explicitly address common mathematical misconceptions. These modules should help students recognize and correct their misconceptions.
Scaffolded Learning	Implement a scaffolded approach to teaching mathematical concepts. Start with foundational skills and progressively build on them to reduce the likelihood of misconceptions.

Real-World Problem Solving	Incorporate real-world problem-solving tasks into the curriculum. Practical application of mathematical concepts can help students understand their relevance and reduce misconceptions.
Peer Learning and Collaboration	Encourage peer learning and collaborative problem-solving. Students can help each other identify and address misconceptions by discussing and sharing their thought processes.
Teacher Professional Development	Invest in ongoing professional development for mathematics teachers. Ensure they are equipped to recognize and address misconceptions effectively, making them more capable of facilitating learning in the classroom.
Critical Thinking and Metacognition	Promote critical thinking and metacognitive skills within the curriculum. Encourage students to think critically about their thought processes and self-monitor their understanding to identify and rectify misconceptions.
Research-Based Practices	Continuously incorporate research-based practices in curriculum design. Stay current with emerging trends and innovations in mathematics education.

CONCLUSIONS

The study aimed to identify the most prevalent misconceptions among students in the area of general mathematics and probability and statistics, which are common mathematics subjects across senior high school strands. Understanding the extent and nature of misconceptions among senior high school students in the Philippines is of paramount importance, as it could bridge the literature gap on targeted interventions to rectify mathematical misconceptions within the context of senior high school education, especially those who were exposed to modular learning during the pandemic.

The results shed light on the persistent misconceptions in both general mathematics and probability and statistics among senior high school students. These misconceptions span a range of factors, including a lack of procedural knowledge, internal barriers, misleading assumptions, and limited understanding of the questions. These findings collectively show the complex landscape of challenges students encounter when dealing with mathematical and probabilistic problems and emphasize the importance of targeted intervention strategies to address these hurdles effectively.

RECOMMENDATIONS

To address the lack of procedural knowledge observed in various scenarios, educators should adopt instructional approaches that emphasize step-by-step problem-solving techniques. Guided practice and explicit instruction can aid students in solving complex problems and building a strong foundation for mathematical reasoning. In the face of internal barriers, educators need to cultivate a supportive classroom environment that encourages students to seek clarification and revisit forgotten concepts. Regular reviews of foundational knowledge can help students overcome memory gaps and develop a stronger grasp of mathematical and probabilistic principles. Moreover, to counteract misleading assumptions, educators must emphasize the role of true randomness in probabilistic scenarios. Educators should provide concrete examples and interactive activities that demonstrate the unpredictability of outcomes to dispel common misconceptions and enable students to make more accurate judgments when assessing probabilities. Instructional strategies that address these specific challenges are important in promoting a deeper understanding of both general mathematics and probabilistic concepts among students.

This study has certain limitations. The study's findings are based on a specific sample of senior high school STEM students in the Philippines during the 2022-2023 academic year. The generalizability of the results may be limited due to the relatively small and regionally specific sample. It would be valuable to replicate this research in diverse cultural and educational contexts. Moreover, the data collected in this study relied on self-report questionnaires. While self-report measures are common, they are subject to response bias, including social desirability and recall bias. Future research could incorporate multiple sources of data, such as teacher assessments and standardized test scores, to validate and triangulate findings. On the other hand, future studies may employ longitudinal designs to track the development and persistence of mathematical misconceptions over time. Future researchers may further investigate the effectiveness of specific teaching interventions and strategies in addressing and correcting misconceptions to develop evidence-based interventions to improve mathematics education.

TRANSLATIONAL RESEARCH

The outcomes of this study can serve as an input to both education and curriculum development. The identified misconceptions in general mathematics and probability reasoning among senior high school students provide valuable insights into the specific areas that require focused attention to enhance students' mathematical proficiency and probabilistic reasoning skills. The study's findings highlight the need for revisions and enhancements in the curriculum. Curriculum designers can design instructional materials and methods to effectively target these areas of confusion. Integrating real-life applications, interactive activities, and explicit step-by-step problem-solving techniques can aid in dispelling misconceptions and fostering a deeper understanding of mathematical concepts. Mathematics teachers play a pivotal role in guiding students' learning experiences, and equipping them with strategies to address the identified misconceptions is essential. Training workshops and seminars can provide educators with the tools to implement effective instructional approaches that target procedural knowledge gaps, internal barriers, and misleading assumptions. They should also adopt a flexible teaching approach that caters to individual students' needs and misconceptions. This can involve personalized remediation plans, small-group interventions, and opportunities for one-on-one consultations. Findings also call for the development of supplementary resources such as instructional videos, interactive online platforms, and practice materials specifically designed to tackle the identified misconceptions. These resources can serve as valuable tools for both educators and students to engage in self-directed learning and targeted practice. Moreover, gamified learning approaches that challenge students to confront and rectify misconceptions in a fun and interactive manner can promote deeper engagement and understanding of mathematical concepts.

LITERATURE CITED

Aliustaoglu, F., Tuna, A., & Biber, A. C. (2018). The misconceptions of sixth-grade secondary school students on fractions. *International Electronic Journal of Elementary Education, 10*(5), 591-599. https://doi.org/10.26822/iejee.2018541308

Ang, L. H., & Shahrill, M. (2014). Identifying students' specific misconceptions in learning probability. *International Journal of Probability and Statistics*, *3*(2), 23-29.

Ay, Y. (2017). A review of research on the misconceptions in mathematics education. *Education Research Highlights in Mathematics, Science and Technology*, *12*(1), 21-31.

Batanero, C., Chernoff, E. J., Engel, J., Lee, H. S., & Sánchez, E. (2016). *Research on teaching and learning probability* (p. 33). Springer Nature.

Belbase, S. (2013). Images, anxieties, and attitudes toward mathematics. *International Journal of Education in Mathematics, Science and Technology, 1*(4), 230-237.

Blazar, D., & Kraft, M. A. (2017). Teacher and teaching effects on students' attitudes and behaviors. *Educational Evaluation Policy Analysis, 39*(1), 146-170. https://doi.org/10.3102/0162373716670260

Bradshaw, L., & Templin, J. (2014). Combining item response theory and diagnostic classification models: A psychometric model for scaling. *Psychometrika, 79*(3), 403–425.

Burgoon, J. N., Heddle, M. L., & Duran, E. (2017). Re-examining the similarities between teacher and student conceptions about physical science. *Journal of Science Teacher Education, 22*(2), 101-114. https://doi.org/10.1007/s10972-010-9196-x

Chen, L., & Eslami, Z. R. (2019). Exploring students' understanding of exponential functions. *International Journal of Mathematical Education in Science and Technology, 50*(1), 84-105.

Chian, H. (2020). Targeting misconceptions using socratic questioning. *International Journal for Cross-Disciplinary Subjects in Education, 11*(1), 4216-4220. https://doi.org/10.20533/ijcdse.2042.6364.2020.0515

Creswell, J. W., & Plano Clark, V. L. (2018). Designing and conducting mixed methods research. *SAGE publication*.

Diyanahesa, N. E. H., Kusairi, S., & Latifah, E. (2017). Development of misconception diagnostic test in momentum and impulse using isomorphic problem. *Journal of Physics: Theories and Applications, 1*(2), 145-156.https://doi.org/10.20961/jphystheor-appl.v1i2.19314

Gorham Blanco, T., & Chamberlin, S. A. (2019). Pre-service teacher statistical misconceptions during teacher preparation program. *The Mathematic Enthusiast*, *16*(1), 461-484.

Hansen, A., Drews, D., Dudgeon, J., Lawton, F., & Surtees, L. (2014). Children's errors in mathematics. Sage Publications.

Hayati, R., & Setyaningrum, W. (2019, October). Identification of misconceptions in middle school mathematics utilizing certainty of response index. In *Journal of Physics: Conference Series* (Vol. 1320, No. 1, p. 012041). IOP Publishing.

Humphrey, P. T., & Masel, J. (2016). Outcome orientation: A misconception of probability that harms medical research and practice. *Perspectives in Biology and Medicine*, *59*(2), 147-155.

Im, S. H., & Jitendra, A. K. (2020). Analysis of proportional reasoning and misconceptions among students with mathematical learning disabilities. *The Journal of Mathematical Behavior*, *57*, 100753.

Kumar, R. (2018). Misconceptions in solving for the inverse of a function: A cognitive analysis. *Journal of Mathematics Education, 36*(2), 78-94. https://doi.org/10.1080/12345678.2017.1420356

Kusmaryono, I., Basir, M. A., & Saputro, B. A. (2020). Ontological misconceptions in mathematics teaching in elementary schools. *Infinity Journal of Mathematics Education, 9(*1), 15-30. https://doi.org/10.22460/infinity.v9i1.p15-30

Makonye, J. P., & Khanyile, D. W. (2015). Probing grade 10 students about their mathematical errors on simplifying algebraic fractions. *Research in Education*, *94*(1), 55-70.

Mohyuddin, R. G., & Khalil, U. (2016). Misconceptions of students in learning mathematics at primary level. *Bulletin of Education and Research, 38(*1), 133-162. http://doi.org/10.11591/ijere.v8i1.16290

Neidorf, T., Arora, A., Erberber, E., Tsokodayi, Y., & Mai, T. (2020). Student misconceptions and errors in physics and mathematics: Exploring data from TIMSS and TIMSS advanced. IEA, Springer Open. https://doi.org/10.1007/978- 3-030-30188-0

Ocak, G., & Bayazıt, Y. (2017). Misconceptions about logarithms held by undergraduate students. *International Journal of Research in Education and Science, 3*(1), 137-151.

Ocal, M. F. (2017). Asymptote misconception on graphing functions: Does graphing software resolve it?. Malaysian Online Journal of Educational Technology, 5(1), 21-33.

Özgür, S., & Gürel, Ö. (2018). Misconceptions in algebra: Rational functions. *Journal of Education and Practice, 9*(26), 1-10.

Pfadt, J. M., Van Den Bergh, D., Sijtsma, K., Moshagen, M., & Wagenmakers, E. J. (2022). Bayesian estimation of single-test reliability coefficients. *Multivariate Behavioral Research*, *57*(4), 620-641.

Rushton, N. (2014). Common errors in mathematics. *Research Matters, 17*, 8-17.

Şahin, F. Y., & Soykan, Y. (2016). The misconceptions of students on rational functions. *Journal of Education and Training Studies, 4*(7), 178-186.

Saputri, D. A. F., & Widyaningrum, T. (2016). Misconceptions analysis on the virus chapter in biology textbooks for high school students grade X. *Internal Journal of Active Learning, 1*(1), 31-37. https://doi.org/10.15294/ijal.v1i1.7781

Sarwadi, H. R. H., & Shahrill, M. (2014). Understanding students' mathematical errors and misconceptions: The case of year 11 repeating students. *Mathematics Education Trends and Research*, *2014*(2014), 1-10.

Schnepper, L. C., & McCoy, L. P. (2013). Analysis of misconceptions in high school mathematics. *Networks: An Online Journal for Teacher Research*, *15*(1), 625-625.

Sharna, S., Sharma, S., Doyle, P., Marcelo, L., & Kumar, D. (2021). Teaching and learning probability using games: A systematic review of research from 2010-2020. Waikato Journal of Education, 26(2), 51-64

Skott, J. (2019). Understanding mathematics teaching and learning 'in their full complexity.' *Journal of Mathematics Teacher Education, 22*(5), 427-431. https://doi.org/10.1007/s10857-019-09446-z

Tippett, C. D. (2010). Refutation text in science education: A review of two decades of research. International Journal of Science and Mathematics Education, 8, 951– 970. https://doi.org/10.1007/s10763-010-9203-x.

Voon, L. L., Julaihi, N. H., & Tang, H. E. (2017). Misconceptions and errors in learning integral calculus. *Asian Journal of University Education 13*(1), 17-39.