Vol. 4 · July 2014
Print ISSN 2244-1824 · Online ISSN 2244-1816
doi: http://dx.doi.org/10.7719/irj.v4i1.304

JPAIR Institutional Research is produced
by PAIR, an ISO 9001:2008 QMS certified
by AJA Registrars, Inc.

# Equations and Formulas as Problem Solving Strategies Frequently Used by Students in Solving Math Problems

**SALVADOR A. LORIA JR.**
http://orcid.org 0000-0002-0215-2687
buddyloria2000@gmail.com
Nueva Ecija University of Science and Technology
Nueva Ecija, Philippines

## ABSTRACT

Problem solving is the core and primary objective of mathematics teaching. This study aimed to describe the problem solving abilities of sophomore engineering students of Nueva Ecija University of Science and Technology (NEUST) through the use of case study type, qualitative research method. Results showed that in terms of the ability to identify goals in a problem, the process of problem solving, and the skills of students in problem solving, the respondents showed a satisfactory performance. The most common method used by the respondents in problem solving was the use of equations or formulas, while the least preferred methods were the strategy of working backward and logical reasoning. The respondents displayed difficulty in answering problems in motion, geometry and set operation while the problems about sequence, rate, age, money/investment, combination, time were found easy to solve. Teachers and educators of mathematics are advised to develop instructional materials and activities that will equally address the three different context of problem solving, namely, goal of the problem, process in problem solving, and problem solving skills of students.

***Keywords -*** Mathematics Education, problem solving, abilities, goal, process, skill, qualitative method, Nueva Ecija, Philippines.

## INTRODUCTION

The profound and rapid changes in the world today brought about by the advancement in knowledge and technology signal the need for education to keep abreast with these changes to be continuously relevant and functional. Today's education needs to strengthen the teaching of Science and Mathematics because it is believed that these fields serve as a gateway toward industrial globalization.

Mathematics plays a vital role in an individual's everyday life. Everyone needs to understand the power and beauty of mathematics and be able to use it for practical and future use, both personal and occupational. Students have to be provided with mathematics education that will enable them to fulfill their personal ambitions and career goals. It is for this reason that instructional focus needs to be redefined so that students' learning will serve them well throughout their lives.

As claimed by the National Council for Teachers in Mathematics NCTM (1980), problem solving is an integral component of essential mathematics and is the principal reason for studying it. It should be integrated throughout all courses and all grade levels (Edwards, Hatfield & Bitter, 2000). There are many benefits of introducing students to problem solving early and consistently. Problem solving integrates all areas of the curriculum because it draws on reading, writing, social studies, economics, and science when developing real-world examples. How to decode the problem requires reading skill and how to translate the answer to a meaningful end needs a writing skill. The subjects of problems can convey information about nearly any topic, and students can learn a variety of facts while they are applying strategies to solve problems. Problem solving equips students with skills to reason logically and to think in an abstract or formal ways since it is more than just a memorized activity. It deduces students' interest and enthusiasm and motivation (NCTM, 1989). More importantly, problem solving is an effective means in making mathematics more relevant to students.

Problem solving can be interpreted in different contexts: as a goal, as a process, and as a skill (Edwards, Hatfield & Bitter, 2000). Problem solving as a goal can be described as identifying and attaining the desired end, regardless of methods and procedures, type of problems or even mathematical content. The student's ability to identify the unknown of the problem early has a big impact to his success to carry on. If a student cannot think about his goal before attacking the problem, this shows he does not fully understand the problem, and, therefore, succeeding efforts may end up futile.

Problem solving as a process beholds as an opportunity to exercise methods, strategies, and heuristics planned and executed to attain the goal of the problem. Polya's (1957) process of solving problems uses four phases: 1) understanding the problem; 2) devising a plan for the solution; 3) carrying out the devised plan; and 4) verifying the obtained answer. In the process, it also involves the discovery of strategies and solutions on one's own. In other words, self-invented strategies may crop up along the way. The strategies used to solve a problem are determined by two factors: first, the competence and sophistication level of the student; and second, the mathematical tools that the student has previously mastered (Artzt & Armour-Thomas, 1992). The more complex the problem to be solved, the more strategies may be required to solve the problem.

Problem solving as a skill is interpreted to particular type of problems and methods of solution. Carr and Jessup (1999) hold that the best way to develop and refine skills of students in problem solving is simply to practice solving a variety of problems. Problems are commonly classified into different types: open-ended, closed, discovery and guided discovery. The open-ended types of problems have a number of possible solutions. Discovery types of problems normally have terminal solution, but there are varieties of approaches a student can use to reach a solution. Guided discovery types include clues and even directions for solving the problem. Closed types of problems follow a well-known pattern of solution and encourage memorization of typical methods. Jonassen (2003) claims that one dilemma in problem solving is that students are mostly taught with well-structured problems that are quite different from actual encountered situations. Typically, these are the types of problems in textbook exercises that require the application of algorithms and memorized routine methods. As a result, students are mostly unable to transfer the skills that they do develop to novel problem situations. He added that oftentimes, particularly in employment settings, the types of problems that they may encounter are non-routine and require creativity and discreet reasoning to solve. If students work on similar problems repeatedly, they may get bored, while when introduced with new types of problems, they continue to feel challenged. These premises suggest that students must have sound experiences of problem solving in the encompassment of goal, process, and skill to become more proficient problem solvers.

When examined as a whole, problem solving as seen within the framework of goal, process, and skills can be rolled into one. The goal sets the process to be accomplished. When the process in problem solving is repeatedly and correctly achieved, then it becomes a skill.

## FRAMEWORK

Problem solving is more than acquiring answers. It is an instrument of developing logical and reasoning skills, a vehicle of thinking, and a philosophy. It is an opportunity to learn the most that can be accumulated from experiences. Problem solving is mainly an avenue of thinking, investigating a situation, using logic and analytical skills that are not learned through regular rote of precise facts. It is an engaging and placing of oneself in the problem solving process and exercising both preceding experience and knowledge to the problem at hand (Sari, 2008).

According to Hatfield, Edwards & Bitter (2000), problem solving can be interpreted in three (3) different contexts: as a goal, as a process, and as a skill. From these premises, the framework of the study was developed.

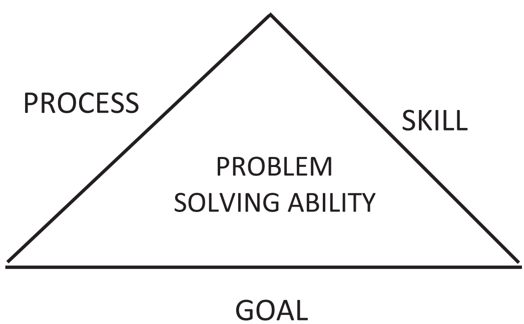


Figure 1. Conceptual Framework of the Study

The study adopted the theory of the equilateral triangle where each side has equal length. In the model, the three sides represent the goal, process and skill in problem solving and the area bounded by these three sides represents the problem solving ability of the student. In calculus, it is proven that the area of any triangle is maximum when it is equilateral. This concept, when related to problem solving, implies that the ability in problem solving can be fully developed when these views of problem solving will be given equal importance and emphasis. Lesser emphasis in any of them means that the problem solving abilities of students are not developed to the fullest. It is equivalent to say that students can become and can be developed to be expert problem solvers when they are trained equally in the three context of problem solving.

The goal of the problem is identifying and attaining the desired end or what the problem asks. If a student cannot identify what he is supposed to attain in the end, succeeding efforts will just be attributed to guessing or taking chances. Identifying the unknown of the problem early would certainly lead to successful and productive effort. This simply means it directs the student to the right track of the road, leading to the solution of the problem.

Problem solving as a process is a planned strategy to attain the goal of the problem. The strategy can be outlined correctly by looking for patterns or relationships among the given conditions of the problem. It can also be strategized using table or list of the given data, drawing pictures or diagram to have a visual perception and representation of information, making use of logical reasoning, guessing and checking the probable answer, using of algebraic equations or formulas, or even working backwards.

Skill as one of the cited factors to determine solution strategies, suggests that it is closely related to the other views of problem solving. Students develop the skill in problem solving when they can solve different types of problems and apply different methods of solution. As claimed by Carr and Jessup (1999), the best way to develop this is to practice solving variety of problems with varied solutions.

Typically, problems given to students in the classroom are routine ones; that is, problems that require the application of algorithms and memorized procedures. These are the types appearing mostly in the textbook exercises. In this regard, it is important to expose students on problems related to occupational settings or real-life situation so that problem solving becomes more than a procedural activity. From non-routine problems, they can think of the novel and creative strategies that make sense to them. In this way, they become continuously motivated to invent strategies that will challenge them at all times.

When viewed as a whole, problem solving within the context of goal, process, and skill can be rolled into one. The goal sets the process to be pursued. When the process is done repeatedly and correctly, it becomes a skill. When all these three views of the problem solving are developed to the fullest; institutions of higher learning which offer technological courses like engineering have succeeded in training and producing graduates who will move the development of the country. These premises and concepts guided the researcher in conducting this study.

## OBJECTIVES OF THE STUDY

This study determined the abilities of the sophomore engineering students in solving Math problems. Specifically, it sought to describe students' problem solving abilities in terms of interpreting the goal of the problem, the process in problem solving and their skill in problem solving. It also describes the frequency of strategies that students would be employed in solving different types of problems and the problem types which they would find difficult, moderate, and easy to solve.

## METHODOLOGY

The study utilized the qualitative method of research, the case study type. It aimed at describing the problem solving abilities through different contexts namely: goal, process and skill. Students' abilities in problem solving were described from their work in the given problem solving activities, which were scored objectively through the use of rubrics. Structured interviews were conducted to clarify students' thoughts. A case analysis was done to describe qualitatively how the students determined the goal of the problem; solution strategies that they employed; and their skills in problem solving.

Samples of the study were 15 sophomore engineering students from the Nueva Ecija University of Science and Technology, Philippines. They were chosen purposively on the basis of their ability in mathematics such as high, average and low. It also considered their scores in the admission test and their field of specialization, namely, Civil Engineering (CE), Electrical Engineering (EE) and Mechanical Engineering (ME). The researcher obtained informed consent from everyone who was interviewed on given questions to answer.

The instruments and materials used in gathering data were problem solving activity sheets; rubric as scoring guide; audio and video recorder to record the responses of students; and a camera to document the activities of data gathering. Data gathered were analyzed qualitatively through cross-checking, triangulation of students' solution, and individual interviews with them. Data were interpreted qualitatively using scales.

## RESULTS AND DISCUSSION

### A. Problem Solving Ability as a Goal, Process, and Skill

**1. Goal**. Students described the desired end of the problem in various ways using symbols or variables to represent what is being asked in the problem; copying exactly the goal stated on the problem; and using code or abbreviation. The interrogative words such as how, what, when, and which guided them in determining the goals of the problems. The ability of students in describing the goals of the problems were the following: three students demonstrated excellent ability; four students showed a very satisfactory ability; three students showed satisfactory ability; four students displayed fair ability and one student with low ability. In most cases, the ability of students in determining the goal of the problem was **satisfactory** since 76.95 percent of the problems had been correctly solved by the students through correct identification of goals. This performance indicates that students had complete understanding of the problem situations. This result is supported by the study of Lugo (2011) on the capabilities of First-Year engineering students in solving word problems. She stated that the common mistakes of students were wrong representation, misinterpretation and forgotten concepts. This indicates that understanding the goal of the problem through representation, interpretation and use of conceptual knowledge is pivotal element in problem solving.

**2. Process**. The mean scores of each student were computed to describe their ability as to the process of problem solving. Charles (1987) described the evaluation techniques for effective instruction as holistic and analytic scoring. In this study, rubric holistic scoring guide was used to score objectively the problem solving process presented by the students. There were four out of 15 students who exhibited **very satisfactory** performance as to the process of problem solving. Following the framework of Polya (1957) in solving a problem: the four mentioned students demonstrated complete understanding of the problem; identified all significant elements of the problem and determined its relationship; created appropriate plan and used correct strategy; executed the adopted solution strategy completely and correctly; successful in attaining the correct answer; and verified the correctness and reasonableness of the obtained answer.

Only one student depicted ***not satisfactory*** performance in problem solving process which illustrates that: he had incomplete understanding of the problem situation; he wrongly described the goals of the problems; he devised incorrect plan and strategy; he performed incorrect mathematical computation; and he ended with no answer or his answer was a result of pure guess.

On the average, the 15 participants exhibited ***satisfactory*** performance in terms of their process in problem solving. This means that these students almost completed the problem solving process except in verifying the correctness and reasonableness of the acquired answers. They were not able to develop the habit of verifying the correctness of the answer since most of their solutions ended up with their obtained answer. Students treated problem solving as a linear process and not as a cyclic one as suggested by Polya (1957).

The result was equivalent to the study of Zhu (2003) when he analyzed the general strategies on problem solving written on the textbooks from China, US and Singapore. It revealed that Asian textbooks covered the third stage of Polya's model which is "how to carry out the plan." Likewise, the study of Fan and Zhu (2007) stressed the importance of looking back at the final answer in the illustration of problem solutions. This is equivalent to the last stage of Polya's model which is the verification of the correctness and reasonableness of the obtained answer.

**3. Skill**. Five out of 15 students portrayed **very satisfactory** performance in using their skills in solving the problem. These students solved different types of problems, transfer their knowledge to solve new problems, and able to employ different strategies. As spelled out in the study of Tupas (2012), for the students to develop their problem solving skills, they should be exposed to different routine and non-routine problems. He added that the mathematical concepts and skills that they learned can be used to resolve real life problem situations. On the average, the participants of the study demonstrated a **satisfactory** performance as to the skill in problem solving since they were able to solve 66.48 percent of the problems correctly. The mean score indicates that students need to exert more effort in studying problem solving for them to be prepared and competent in the mathematics board exam of engineering considering that 70 percent is the minimum passing score.

**B. Frequency of Used Strategies**

This section describes the solution strategies employed by the students to different types of problem.

**1. Find a Pattern**. Fourteen students applied the "finding a pattern" strategy to problems on sequence and progression at different extent or degree. Five of them used this strategy *frequently*, that is out five problems of this type, four were solved using find a pattern; four of them *sometimes* used this strategy, means three out of the five problems were solved using this strategy; four of them *seldom* used this strategy, that is two of the five problems were solved using the strategy of find a pattern; one student employed this strategy only *once* and one student did *not use this strategy at all*. Herbert and Brown (1997) elaborated the process of solving problems through looking and recognition of patterns. They concluded that the confidence, attitude and ability in algebraic thinking were increased. They also added that these learned strategies of students served as a solid foundation to build formal knowledge and understanding on Algebra. Similarly, the study of Fan and Zhu (2007) on the examination of different mathematics textbooks underscores the importance of looking for a pattern heuristic in the representation of problem solving procedures.

**2. Making Table or List**. The strategy of "making table or list" was executed by 14 students to problems on motion, rate, combination and permutation. Two of them used frequently the solution of making table or list; two students applied this strategy *sometimes*; seven of them executed this strategy *seldom*; three students utilized this strategy only *once* and one student did *not use this strategy at all.* Suzuki and Harnisch (1995) examined the different types of cognitive strategies in problem solving. Results of their study showed that the frequent strategy adopted by the respondents were listing all numbers, listing all possible area codes, listing all odd numbers and listing all possibilities. These results supported the usefulness of "making list or listing elements strategy" in problem solving.

**3. Working Backward**. The working backward approach was not preferred by the students as their problem solving strategy. Only nine out of 15 students applied this strategy to problems on age, rate, time and miscellaneous types. One of them employed this strategy *sometimes*, three students used this strategy *seldom*, five of them used this strategy *once* and six students *did not use this strategy at all*. According to Ayres and Sweller (1990), when unfamiliar problems are presented to students, they tend to employ working backward or means-ends strategy. Students usually start by working backward from the end-result of mathematical statement to the given conditions. A forward-working procedure could then be used to check the correctness of the obtained answer. In one of the geometry experiments of Ayres and Sweller (1990), they found that means-ends analysis or working backward strategy was most likely to be used in determining the sub-goal, then approaching the main goal of the problem.

**4. Guess and Check**. Fourteen students executed "guess and check" strategy to problems on money, coin, number relation and miscellaneous types. One of them used the solution of guess and check in solving problems *always*; three students used this strategy *sometimes*; eight students used this strategy *seldom*; two students used this strategy *once;* and one student did *not use this strategy at all*.

Capraro et al. (2012) promoted the usefulness of guess and check strategy in an open-ended problems. They also added that pre-service teachers frequently use guess and check strategy in solving problem-triangle task.

**5. Drawing Picture or Making Illustration**. All 15 students had been seen drawing picture or making illustration on their solutions to problems in geometry and miscellaneous types. Eight of them employed this strategy frequently, and the other six students used this strategy *sometimes*, and one student used this strategy *seldom* in solving problems. The paper of Rösken and Rolka (2006) outlined the significant role of illustration and visualization in mathematics learning. In particular, their study focused on the ability of students to interpret visually the problem statement. Also, it was found out that 90 percent of students were able to interpret visually the geometrical definition of integral and 77 percent of students illustrated the positive area bounded by the functions.

**6. Use of Logical Reasoning**. The strategy of logical reasoning was not oftentimes seen on the solution of students. Eleven students used this approach to problems on set operation, average, geometry, digit and number relation. One of them frequently applied this approach; two of them administered this strategy *sometimes*; two students used this strategy *seldom*; six students executed this strategy only once and four (4) students *did not use this strategy at all*. Nunes et al. (2007) supported the use of logical reasoning in solving math problems. They claimed that students trained in logical reasoning made more progress compared to the control group who were not subjected to training. They added, students' logical abilities predict their mathematical accomplishment. In a study conducted by Wavering (1989), he emphasized the necessity of logical reasoning to construct line graphs and making illustrations with complete information of the relationship among variables. This is an indication that logical reasoning is a prior skill needed to execute other problem solving strategies.

**7. Use of Equations and Formulas.** This strategy is the most preferred by the students in solving different types of problems. All 15 students employed equations and formulas to problems on age, investment, mixture, coin and digit. Nine of them used this strategy *always*, five students used this strategy frequently, and only one student used this strategy *sometimes* According to the study conducted by Fan and Zhu (2007), the use of equation is one of the noteworthy heuristics presented in the mathematics textbooks of China, Singapore, and US. They are using letters to represent the unknown variables and formulating equations or inequalities based on the problem conditions. This indicates that even foreign mathematics textbooks are dominated by equations and formulas.

Majority of the students always preferred to use formulas and equations in solving problems, meaning out of five problems of different types were solved using this approach. Students sometimes employed the strategy of drawing picture and finding a pattern; they seldom executed for the methods of making table or list and guess and check in solving different types of problems, while the strategy of working backward and logical reasoning in solving problems were used minimally. This implies that students employed these strategies only once. Similarly, the problem solving strategies utilized by the respondents were the same as mentioned in the study of Tupas (2012), which includes guess and test, use of algebraic equations, listing of elements, use of illustrations or diagrams, acting it out and working backwards. Moreover, in the book of O'Connel (2000), the different strategies including make a table, guess and check, use of logical reasoning, work backwards, draw a picture or diagram, find a pattern, and choose an operation were stipulated.

**C. Problems as to Degree of Difficulty**

The motion, geometry and set operation types of problems were found very difficult to solve. Only one student solved the problem on motion, none of the students solved a problem on geometry, while only three students solved a problem on set operation. During the cross checking of activity sheets and interview with the students, their frequent reasons for failing in answering these problem types were wrong identification of goals of the problems and the incorrect use of strategy.

In one of the problems under motion, most of the students thought that the goal of the problem is the time needed by the faster car to catch up the slower car instead of getting the total time until the two cars met and traveled with the same length of distance. In terms of the strategy that students employed to answer this type of problem, most of them used equations and formulas instead of making table. Similarly, in one of the problems under geometry, students were confused to the dimensions of triangle. They thought that the bigger dimension gives larger area. Students were not able to discern the *ambiguous case* of a triangle because they had not drawn or illustrated the correct dimensions. This result supports the study conducted by Hegarty and Kozhevnikov (1999) that the visual-spatial representation is highly correlated with the success of mathematics teaching. They emphasized the importance of visual imagery in analyzing the condition and situation of the problem.

In the "set operation" problem type, students revealed that it was their first encounter with such kind of problem and that they were not also aware of the concept of *Venn Diagram*. On the "average" problem, students had not realized that to get the lowest possible score of one test, the other four tests must have a perfect score of 100. Logical reasoning is necessary for this kind of problem. The same dilemma was encountered by Pollatsek et al. (1981) when they conducted their study on the conceptual understanding of the mean or average. Surprisingly, large number of students was not able to answer this problem type correctly.

The types of problem that the students found moderately difficult to solve are those on progression, digit, mixture, coin and permutation. Eight problems were found moderately difficult to solve since seven to nine students answered them correctly.

In the progression problem, students treated it as moderately difficult to solve because the sequence or pattern can be easily determined which guided them to get the correct answer. In some cases, students were confused with the goal of the problem. In the two problems under progression type, they thought the goal was the "additional blocks" instead of the "total blocks;" "total passengers" instead of the passengers only at $6^{th}$ stop. Similarly with the digit problem, some students were confused with the statement "two-digit number;" they thought of it as two numbers, instead of one number with tens and units digit. Wood and Sellers (1996) stated that solving problems in different ways or using non-procedural method increase conceptual understanding and numerical proficiency of students. Results also revealed that significant difference exist in arithmetic learning of students on problem involving sequence and progression.

In the permutation problem, students were confused with the problem condition "no pose is repeated" for they interpreted it that the three children must change their position for every pose. The same outcome to the study of Rohrer and Taylor (2007) when students given a problem to calculate the number of unique orders or permutations of a letter sequence. It was found out that none of the respondents answered this type of problems correctly. In the coin problem, they interpreted the given condition as "twice as many P10 coins than P5 coins" to mean "the number of P5 coins is twice as much as the number of P10 coins." Misconception on this problem type was also stipulated in the book of Moses et al. (1993) that one coin problem can generate two or more problems by interchanging the given conditions, which is exchanging the number of pieces of coin in the given denominations. In the mixture problem, some students did not know how to get the amount of alcohol content from the given mixture. These were some reasons why students found these problems moderately difficult to solve.

Eight problems were found easy to solve by 10 to 12 students who solved them correctly. These problems were on number relation, money, age, investment and some miscellaneous types. There were twelve problems that were found very easy to solve by the students because 13 to 15 of them answered correctly. Those problems were on combination, time and some miscellaneous types. During the interview, students said that the structure, contents and the conditions of the problems were easy to follow and analyze. Likewise, the solution strategies to these problems were easy to perform and execute. These results affirmed the study of Lawson (2002) when he emphasized the coordination of problem types and problem solving strategies. He concluded that students should be exposed to different routine and non-routine types of problems because this would serve as an avenue to explore various problem solving strategies.

## CONCLUSIONS

The abilities of students in solving mathematics problems are affected by three different contexts of problem solving namely: 1) goal of the problem, 2) process of problem solving, and 3) students' skill in problem solving. The results showed that the frequent reasons in failing to answer math problems correctly are wrong determination of goals and wrong choice of appropriate strategy. In the process of problem solving, students were not able to complete the cyclic process as suggested by Polya. They had not developed the habit of verifying the correctness of answers by looking back at the problem and its conditions since most of their solutions ended at their obtained answers. Mathematics Teachers are encouraged to give their students varied problem solving activities and bring them in their highest ability and creativity in solving problems. They are also advised to look into their teaching strategies so that problem solving can be taught more effectively. Likewise, they are suggested to develop instructional materials that will enhance the abilities of students in interpreting the problem's goal, in the process of problem solving, and in developing problem solving skills. In solving problems, students preferred to employ the strategies of equations and formulas. They have not been exposed to non-procedural or non-algorithmic solutions. Students should be encouraged to solve problems using other solution strategies that make sense to them. They are also advised to go beyond routine solution and algorithm and share their constructed solutions to the class for the benefit of all. The students had difficulty in geometry, motion, and set operation problem types. These problems require reasoning, comprehension, analytical and visualization skills. Mathematics teachers may give enough time and emphasis in teaching geometry and motion problem types. They may also encourage students to employ non-routine strategies in solving different types of problems.

## TRANSLATIONAL RESEARCH

The study shows the importance of the three different contexts of problem solving in teaching mathematics. This would serve as a framework of mathematics educators to design and develop an instructional material and activity that will strengthen the capabilities of their students. This would help the engineering students to develop mathematical and problem solving skills by exercising non-routine strategies and to facilitate them to create self-own strategies. Likewise, the results of the study can be used by the administrators to formulate policies that would improve the educational setting of the university. Moreover, this study can be used by the curriculum developers and planners to explore problem solving in other disciplines.

## LITERATURE CITED

Artz, A. F., & Armour-Thomas, E. (1992). Development of a cognitive-metacognitive framework for protocol analysis of mathematical problem solving in small groups. *Cognition and instruction*, 9(2), 137-175.

Ayres, P., & Sweller, J. (1990). Locus of difficulty in multistage mathematics problems. *The American Journal of Psychology*, 167-193.

Capraro, M. M., An, S. A., Ma, T., Rangel-Chavez, A. F., & Harbaugh, A. (2012). An investigation of preservice teachers' use of guess and check in solving a semi open-ended mathematics problem. *The Journal of Mathematical Behavior*, *31*(1), 105-116.

Carr, M., Jessup, D. L., & Fuller, D. (1999). Gender differences in first-grade mathematics strategy use: Parent and teacher contributions. *Journal for research in mathematics education*, 20-46.

Charles, R. (1987). *How To Evaluate Progress in Problem Solving*. National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 22901.

Fan, L., & Zhu, Y. (2007). Representation of problem solving procedures: A comparative look at China, Singapore, and US mathematics textbooks. *Educational Studies in Mathematics*, *66*(1), 61-75.

Hatfield, M., Edwards, N. T., Bitter, G. G., & Morrow, J. (2000). Mathematics methods for elementary and middle school teachers. *AMC*, *10*, 12.

Hegarty, M., & Kozhevnikov, M. (1999). Types of visual–spatial representations and mathematical problem solving. *Journal of Educational Psychology*, *91*(4), 684.

Herbert, K., & Brown, R. H. (1997). Patterns as tools for algebraic reasoning. *Teaching Children Mathematics*, *3*, 340-345.

Jonassen, D. (2003). Using cognitive tools to represent problems. *Journal of research on Technology in Education*, *35*(3), 362-381.

Lawson, J. (2002). Hands-On Problem Solving, Grade 2 - Page 5. Retrieved on December 18, 2014 from https://books.google.com.ph/books?id=i9xkuQmlrIMC&pg=PA5&dq=problem+types&hl=en&sa=X&ei=nQ2SVKPbKOa7mgWG3ICQBg&ved=0CD4Q6AEwBg#v=onepage&q=problem%20types&f=false

Lugo, M. D. (2011). Correlates of Word Problem Solving Capabilities in Algebra of the First Year Engineering Students. *JPAIR Multidisciplinary Research*, *6*(1).

Moses, B. M., Bjork, E., & Goldenberg, E. P. (1993). Beyond problem solving: Problem posing. *Problem posing: Reflections and applications*, 178-188.

National Council of Teachers of Mathematics (NCTM). (1980). *An Agenda for Action: Recommendations for School Mathematics of the 1980s,* Reston, Virginia: NCTM.

National Council of Teachers of Mathematics (NCTM). (1989). *Curriculum and Evaluation Standards for School Mathematics,* Reston, Virginia: NCTM

Nunes, T., Bryant, P., Evans, D., Bell, D., Gardner, S., Gardner, A., & Carraher, J. (2007). The contribution of logical reasoning to the learning of mathematics in primary school. *British Journal of Developmental Psychology*,*25*(1), 147-166.

O'Connell, S. (2000). *Introduction to Problem Solving: Strategies for the Elementary Math Classroom*. Heinemann, 88 Post Road West, PO Box 5007, Westport, CT 06881.

Pollatsek, A., Lima, S., & Well, A. D. (1981). Concept or computation: Students' understanding of the mean. *Educational Studies in Mathematics*,*12*(2), 191-204.

Polya, G. (1957). *How to Solve It: a new aspect of mathematical method, ed*. London: Penguin.

Rohrer, D., & Taylor, K. (2007). The shuffling of mathematics problems improves learning. *Instructional Science*, *35*(6), 481-498.

Rösken, B., & Rolka, K. (2006). A picture is worth a 1000 words–the role of visualization in mathematics learning. *International Group for the Psychology of Mathematics Education*, *4*, 457

Sari, Y. (2008). Strategi Problem Solving Dalam Pengajaran Matematika.

Suzuki, K., & Harnisch, D. L. (1995). Measuring Cognitive Complexity: An Analysis of Performance-Based Assessment in Mathematics.

Tupas, S. V. (2012). Effectiveness of Problem-Based Learning Approach to the Students' Problem Solving Performance. *JPAIR Multidisciplinary Research*,*9*(1).

Wavering, M. J. (1989). Logical reasoning necessary to make line graphs.*Journal of Research in Science Teaching*, *26*(5), 373-379.

Wood, T., & Sellers, P. (1996). Assessment of a problem-centered mathematics program: Third grade. *Journal for research in mathematics education*, 337-353.

Zhu, Y. (2003). Representations of problem solving in China, Singapore and US mathematics textbooks: a comparative study.